Time-Dependent Analysis to Predict Closure in Salt Cavities

Keshavan Nair
C-Y Chang
R. D. Singh and
A. M. Abdullah
Woodward-Lundgren & Associates
Oakland, California

ABSTRACT

The closure that occurs in conventionally-mined cavities and in solution-mined cavities in salt which are to be utilized for storage is a significant design problem. Furthermore, the surface subsidence that is likely to occur, due to such closure will be significantly influenced by the time-dependent response of the salt. Therefore, it is necessary to develop analytical techniques which can be utilized to predict the closure in salt cavities. The finite element technique is a very powerful tool to solve boundary value problems. A comprehensive computer program which allows for elastic, elastic-plastic, no tension and jointed materials and creep behavior has been developed. The program is capable of solving plane strain and axisymmetric problems. The creep strains are calculated by the power law:

$$\epsilon_e^c = k\sigma_e^n t^m$$

Incremental as well as iterative analyses were used to calculate non-steady creep strains. In these analyses, short time intervals are uneconomical and long intervals lead to instability in solution. Different criteria to overcome this difficulty were examined. After steady state has been achieved, further deformation is computed directly. Illustrative boundary value problems involving elastic and elastic-plastic materials with creep characteristics under plane strain and axisymmetric conditions were solved, and the results compared with theoretical solutions. Excellent agreement between the two was observed. Utilization of these analytical techniques to practical problems is discussed.

INTRODUCTION

The crust of the earth is in a complex state of stress. Underground excavations in this stressed medium change the existing stress conditions in rock surrounding an opening. The changed stress pattern results, in general, in elastic and inelastic displacements. As a result of these displacements, a further distribution of stresses takes place. In materials like rock salt, potash, etc., these displacements will be time-dependent. The closure that occurs with time in cavities created for storage reduces the usable volume. The subsidence over mined areas in rock salt is also a time-dependent phenomenon. Sometimes, creep may result in the transfer of load to weaker stratum over mined areas resulting in progressive failure which, if allowed to propagate, can result in caving and a catastrophic type of failure. Changes in the shape of openings with time and rock bursts are the evidence to indicate that effects of time on stress distribution and deflection can be significant. Traditional analytical methods cannot take into account all the factors affecting the magnitude and nature of deformations of salt cavities and have to make simplifying assumptions. Even with these assumptions, only simple problems can be solved. The availability of high speed computers and numerical techniques such as the finite element technique, have greatly reduced the number of simplifying assumptions necessary for analysis.

The present paper describes the modification of a program developed by the authors (Chang and Nair, 1972) to take into account time-dependent material characteristics. This permits the prediction of closure and subsidence as a function of time. First the choice of model to represent the salt is described. Then the method of analysis is outlined and finally the results of illustrative examples are presented.

CHOICE OF A MATERIAL MODEL

The choice of a suitable mathematical model to represent the time-dependent behavior of a material is an essential and important step in the solution of a creep problem. The selection of a model is based on its ability to predict the behavior of the real material. The model should be complex enough to duplicate the behavior and simple enough to be handled mathematically.

The theory of linear viscoelasticity has been utilized to analyze time-dependent problems by classical techniques and in conjunction with the finite element method (Ginirk and Johnson, 1964; Taylor and Chang, 1966; White, 1968). However, it has been observed that the time-dependent response is significantly nonlinear; and hence, the linear viscoelastic approach may not be suitable.

Tests conducted by U.S. Army Engineers WES (1963) showed that rock salt is elastic-plastic in nature. It was observed that after confining pressure of 2500 psi, the effect of increase of confining pressure on the strength is not appreciable. On the basis of these results, rock salt can be modeled as Mohr-Coulomb material at low pressure and Von Mises material at high pressures. For the time-dependent response, Boresi and Deere (1963) suggested a creep law of the form:

$$\epsilon_c = k\sigma^n t^m \tag{1}$$

where ϵ_c is the creep strain, σ is the stress difference causing creep in psi and t is the time in hours. Nair (1970), on the basis of additional extension tests, found that this law provided a reasonable representation of the data.

Serata (1968) proposed a viscoelastic-plastic rheological model to represent the response of salt to load. This model requires evaluation of ten coefficients. The evaluation is very cumbersome and requires extensive testing, and it is extremely doubtful if the representation is more accurate than the empirical power law. Winkel et al. (1971) reported that considerable variation in these coefficients can be expected from site to site. Parameters obtained for material obtained at one site cannot be used for another.

In view of these facts, the empirical law of the type presented in Eqn (1) was used in this study. The coefficients k, m, and n can be easily obtained in accordance with the method described by Nair and Deere (1969). It must be pointed out that Equation (1) holds under uniaxial loads. In a multiaxial stress field, σ is replaced by an equivalent stress σ_e.

FORMULATION AND ANALYSIS OF CREEP PROBLEMS

As with any boundary value problem, there are three essential elements in formulating the problem: (i) governing equations (equilibrium and compatibility), (ii) the stress-strain relations, and (iii) boundary conditions. It is then necessary to develop solution techniques to solve the problem as formulated. The governing equations apply to all problems and are described in detail in textbooks on mechanics. The boundary conditions specify the particular problem under consideration. It is the stress-strain relations which specify the material characteristics. These are described in this paper. Following the discussion on stress-strain relations, solution techniques for representative problems based on the finite element method are discussed.

Creep stress-strain relations

General. The creep deformations of isotropic materials are shear deformations which are accompanied by little volume change. The following assumptions are made in the derivation of stress-strain relations during creep

(a) No volume change occurs due to creep strains; i.e., the material is incompressible after creep begins.
(b) The principal shear strain rates are directly proportional to the corresponding principal shear stresses.
(c) The principal strain axes do not rotate under deformation; i.e., the principal normal strains are in the same direction as the corresponding principal normal stresses.
(d) The strains are small.

From assumptions (b) and (c), we can write in general:

$$\frac{\dot\epsilon^c_x - \dot\epsilon^c_y}{\sigma_x - \sigma_y} = \frac{\dot\epsilon^c_y - \dot\epsilon^c_z}{\sigma_y - \sigma_z} = \frac{\dot\epsilon^c_z - \dot\epsilon^c_x}{\sigma_z - \sigma_x} = \frac{\dot\epsilon^c_{xy}}{\sigma_{xy}} = \frac{\dot\epsilon^c_{yz}}{\sigma_{yz}} = \frac{\dot\epsilon^c_{xz}}{\sigma_{xz}} = \phi \tag{2}$$

For our purpose $\dot\epsilon^c_{yz} = \dot\epsilon^c_{xz} = 0$ as $\sigma_{yz} = \sigma_{xz} = 0$ in the boundary problems of interest. Defining equivalent stress as:

$$\sigma_e = \sqrt{\frac{3}{2}} \left\{\sigma'_{ij}\ \sigma'_{ij}\right\}^{1/2} \tag{3}$$

where

$$\sigma'_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\ \delta_{ij} \tag{4}$$

From these equations and the assumption of no volume change

$$\sigma_e = \sqrt{\frac{3}{2}}\ \frac{1}{\phi}\left[(\dot\epsilon^c_x)^2 + (\dot\epsilon^c_y)^2 + (\dot\epsilon^c_z)^2 + 2(\dot\epsilon^c_{xy})^2\right]^{1/2} \tag{5}$$

Defining equivalent strain as

$$\dot\epsilon^c_e = \sqrt{\frac{2}{3}}\left\{\dot\epsilon^c_{ij}\ \dot\epsilon^c_{ij}\right\}^{1/2} \tag{6}$$

Equation (6) can be written as

$$\dot\epsilon^c_e = \sqrt{\frac{2}{3}}\ \left[(\dot\epsilon^c_x)^2 + (\dot\epsilon^c_y)^2 + (\dot\epsilon^c_z)^2 + 2(\dot\epsilon^c_{xy})^2\right]^{1/2} \tag{7}$$

Substituting this value in Equation (5) gives:

$$\phi = \frac{3}{2} \frac{\dot{\epsilon}^c_e}{\sigma_e} \tag{8}$$

From Equation (2) and (8) we get

$$\dot{\epsilon}^c_{ij} = \frac{3}{2} \frac{\dot{\epsilon}^c_e}{\sigma_e} \sigma'_{ij} \tag{9}$$

Equation (9), with the creep law $\epsilon^c_e = k\ \sigma^n_e\ t^m$, is used to get the components of creep strain increment at any time.

Plane strain. Equation (9) is a general equation and can be used for axisymmetric problems. However, when using it for plane strain problems proper consideration should be given to the kinematic constraints in the Z direction.

The stress-strain relationship for a material when creep is continuing can be written as

$$\left.\begin{aligned} \epsilon_x &= \frac{1}{E}\ \{\sigma_x - \nu(\sigma_y + \sigma_z)\} + \epsilon^c_x \\ \epsilon_y &= \frac{1}{E}\ \{\sigma_y - \nu(\sigma_z + \sigma_x)\} + \epsilon^c_y \\ \epsilon_z &= \frac{1}{E}\ \{\sigma_z - \nu(\sigma_x + \sigma_y)\} + \epsilon^c_z \end{aligned}\right\} \tag{10}$$

For plane strain $\epsilon_z = 0$. From the last of Equation (10)

$$\sigma_z = \nu(\sigma_x + \sigma_y) - E\ \epsilon^c_z \tag{11}$$

substituting this in the first two of Equations (10)

$$\left.\begin{aligned} \epsilon_x &= \frac{1}{E}\{(1 - \nu^2)\ \sigma_x - \nu(1 + \nu)\ \sigma_y\} + \epsilon^c_x + \nu\epsilon^c_z \\ \epsilon_y &= \frac{1}{E}\{(1 - \nu^2)\ \sigma_y - \nu(1 + \nu)\ \sigma_x\} + \epsilon^c_y + \nu\epsilon^c_z \end{aligned}\right\} \tag{12}$$

It is easily seen that in Equation (12) the quantities within the brackets are elastic strains. This shows to account for ϵ^c_z in a plane strain problem, a strain equal to $\upsilon\epsilon^c_z$ is added to the strains in x and y directions. To calculate σ_z, Equation (11) is utilized.

It should be recognized that the assumption $\epsilon^c_z = 0$ for plane strain case will not be correct. The total strain and not just the creep strain in the Z direction is zero.

Furthermore, in practical problems of stress analysis, the stresses change with time, and the question arises: What is the strain-time path under varying stress? Nair and Boresi (1970) addressed themselves to this question. Age-hardening or strain-hardening laws can be used. Considering the present level of knowledge with regard to material properties and other in situ phenomena, the difference in the strain-hardening and time-hardening approaches cannot be considered significant (Nair, 1970; Greenbaum, 1966).

Solution technique

Because of the complexity of the boundary value problems associated with practical problems in rock mechanics, it is necessary to utilize numerical techniques to obtain solutions for representative boundary value problems. Of the numerical techniques, the finite element technique has proven to be the most powerful for the solution of boundary value problems (Clough, 1960; Wilson, 1963; Zienkiewicz, 1972).

The basic concepts of the finite element method have been discussed extensively in literature. Various procedures have been utilized in conjunction with this method to include non-linear stress-strain properties, elasto-plastic yielding and joints and the geological discontinuities in the stress analysis of rock masses. Chang and Nair (1972) developed a computer program which has the following features:

1. The tensile stress is not allowed to exceed a prescribed tensile strength. Materials with no tensile strength can be analyzed.
2. No stress point in the stress space is allowed to stay outside the yield surface for the material. The material can be either Mohr-Coulomb or Von Mises.
3. A joint perturbation analysis can be conducted to take into account joints and geological discontinuities. If the normal stress across a joint is tensile, it is assumed that the joint is incapable of resisting any shear stress.

Details of the computer program or the theory behind the analysis will not be repeated here. They are given in Chang and Nair (1972). Only the necessary modification of the program to include time-dependent or creep analysis is outlined below.

Time-dependent analysis for problems in rock mechanics can be considered in two categories. The first includes those cases where the boundary value problem changes with time. Such problems include gradual creation of cavities. The second category includes those problems where the properties of rock are time-dependent. This is essentially the case when materials like rock salt are involved. It is this case which is considered here. Time-dependent analyses have been developed by Boresi and Deere (1963), Greenbaum (1966), Nair (1967), Aiyer (1969), Nair and Boresi (1970), and Winkel et al. (1971). However, these analyses could only solve relatively simple boundary value problems. For example, they could not consider bedding planes, no tension, elasto-plastic behavior, etc. The analysis can be conducted by two procedures: (a) Iterative procedure and (b) Incremental procedure. Computer programs have been developed to utilize either procedure.

Creep Analysis by Iterative Procedure. A method of successive approximation is used to solve the problem. The relationship between stress-strain and time is given by:

$$\epsilon_e = \epsilon_o + k\sigma_e^n t^m$$

$$= \frac{\sigma_e}{E_o} + k\sigma_e^n t^m$$

Defining the time-dependent modulus E(t) as

$$E(t) = \frac{\sigma_e}{\epsilon_e}$$

$$E(t) = \frac{E_o}{1 + E_o k\sigma_e^{n-1} t^m} \quad (13)$$

where:

E_o = Initial modulus

σ_e = Effective stress

ϵ_e = Effective strain

It is assumed that change in modulus with time is such that material remains isotropic. The first computation is made at t = 0. At the next time interval, a new modulus is computed from Equation (13) and a new stress distribution obtained. This is checked for convergence with the previous stress distribution. Convergence may be measured in terms of effective stress. If convergence is not satisfactory, the new stress distribution is used to compute a new modulus for every element and the analysis repeated. After convergence is reached, the analysis moves to the next time interval. There are no convergence problems associated with an iterative procedure; however, there can be significant errors in the results for certain classes of boundary value problems.

Creep analysis by incremental procedure. In this procedure, the total time of analysis is divided into a sequence of time intervals. It is assumed that the stresses remain constant during the interval. The creep strains occurring within the interval can be calculated. The creep strains are distributed elastically to restore compatibility. This method is often referred to as the "Initial Strain" technique or a method of successive elastic solutions. The procedure is described step-by-step below.

a. An elastic-plastic solution which has "no-tension" and joint elements provides the stresses at t = o due to a system of applied loads and boundary conditions. The boundary loads may be applied in steps but any excavation or construction is assumed to have occurred in one step. This has been described in detail by Chang and Nair (1972).
b. The increments of creep strains during a short time interval from t = o to t = Δt are calculated using the creep law $\epsilon_e^c = k\sigma_e^n t^m$. The components of the creep strain increments in case of axisymmetric and plane strain cases are calculated from relations developed earlier.
c. The creep strain increments are treated as initial strains. The initial strains are converted into equivalent nodal point forces and stresses which are added to the applied loadings and an elastic solution is obtained.
d. A check is made to see if, because of the redistribution of stresses, any of the elements is outside the yield surface. If this condition exists, the excess stresses are redistributed as described by Chang and Nair (1972) and the stress condition returned to below the yield level.
e. The method proceeds by taking the second time interval. By keeping record of cumulative stresses and strains, a complete creep solution is obtained. This is continued till the steady state is reached.

Stability of solution. Greenbaum (1966), on the basis of analyzing axisymmetric problems, found if large time increments are used, the solution becomes unstable. The magnitude of the time interval depended on the material constants and the state of stress. Similar problems were experienced during the present study, especially in cases in which creep strains were very large as compared to elastic strains. For economy, such problems have to be solved using as large a time interval as possible within the constraints of solution stability.

During the course of the research described in this paper, a major effort was directed in developing and exploring the methods of automatic generation of time intervals. As the economy and stability of the solution depends upon the time increments, this is discussed in detail in the next section.

Time intervals for stability. As pointed out earlier, to solve problems without spending prohibitive computer time, the time intervals have to be large. But at the same time, they cannot be so large as to cause instability in the solution. In the beginning, i.e., soon after t = o, the stresses change very fast and change little after the steady state has been reached. This means that time intervals have to be smaller initially and may be increased as creep proceeds. For this purpose, an automatic time generation technique becomes indispensible to a computer program of this type. The following methods were tried:

(a) Time Intervals According to Rate of Stress Distribution: This is a technique to increase time interval when the rate of change of stress slows down. The next time interval is calculated based on the maximum allowable change in any interval and the maximum change which occurred during the previous interval. This can be written as

$$\Delta t_{i+1} = t_i \frac{M}{N} \tag{14}$$

where N is the allowable change and M is the change recorded during the previous intervals. Greenbaum (1966) proposed the relationship

$$\Delta t_{i+1} = t_i \cdot \left[\frac{n}{(\Delta\sigma_e / \sigma_e \quad max)}\right] \tag{15}$$

and recommended η as 0.03 to 0.05. It was soon apparent that Equation (15) will increase time intervals too rapidly. For this purpose, another check was applied that

$$\Delta t_{i+1} \leq 1.2 \ t_i \tag{16}$$

and that the maximum change of the effective creep strain must be less than maximum elastic strain.

$$\Delta\epsilon^c_{e_{max}} < \frac{\sigma_{e_{max}}}{E} \tag{17}$$

This corresponds to limiting the maximum time interval so that

$$\Delta t_{max} < \left[\frac{\sigma^{1-n}_{e_{max}}}{Ek}\right]^{\frac{1}{m}} \tag{18}$$

where E is the Young's Modulus, k, n, and m are creep constants.

Treharne (1971) studied this criterion in detail and found that with this criterion, the longest time intervals were reached in some cases soon after arrival at steady state solution. The time intervals decreased and solutions became unstable.

(b) Another criterion is similar to Equation (16) with the additional requirement that the creep strain during an interval is restricted to less than a fraction of the elastic strain at the start of the interval

$$\Delta\epsilon^c_e < \beta\epsilon^c_e \tag{19}$$

This can be written as

$$\Delta t < \frac{\beta}{Ekm\sigma^{n-1} t^{m-1}} \tag{20}$$

The disadvantage of this criterion is that β is not specified. Different values may have to be used for different problems. Also, for steady state condition and for materials m = 1 Equation (20) predicts that Δt will not increase with time.

(c) This criterion was developed by Treharne (1971). It was postulated that difference between true and calculated stresses are in the nature of perturbation. These perturbations attempt to decay. The critical step length is taken as the maximum time step which ensures that the decay of any such perturbation is stable. The slower decay rate, for which the solution is stable, is for a perturbation which is an eigenvector. Therefore, the problem reduced to determining the eigenvector perturbation with the fastest decay rate. This is the lowest eigenvalue. Based on this principle, the following equation was developed for shear perturbations which are most critical.

$$\Delta t < \frac{4}{3} \frac{(1+\nu)}{E} \frac{1}{\dfrac{\partial^2 \epsilon^c_e}{\partial\sigma_e \partial t}} \tag{21}$$

This can be written as

$$\Delta t < \frac{4}{3} \frac{(1+\nu)}{E} \frac{1}{kmn\sigma_e^{n-1} t^{m-1}} \tag{22}$$

Equation (22) is a very simple expression to be used. At this time (1973), with the limited experience, it appears that use of Equation (22) will usually avoid instability in solution and economy in computer time.

Steady state. For some creep problems, a steady-state condition (stress state does not vary with time) is reached after which creep strains and deformations can be extrapolated to the final time of interest without taking any further incremental steps. In the program developed, the steady state was assumed to have been reached when

$$\frac{\Delta\sigma_e}{\sigma_e}_{max} < n_1 \cdot \left(\frac{\Delta\sigma_e}{\Delta t}\right) < n_2 \tag{23}$$

where n_1 and n_2 are constants. After the steady state has been reached, the program extrapolates the displacement and strains.

APPLICATIONS

Elastic thick walled cylinder

An infinitely long, thick-walled cylinder with open ends subjected to internal pressure was selected as an example for axisymmetric body of revolution and in plane strain. The elastic constants used are same as used by Greenbaum (1966). The effective creep strain is assumed to be given by

$$\epsilon_e^c = 6.4 \times 10^{-18}\ \sigma_e^{4.4}\ t \tag{24}$$

and Modulus of Elasticity = 2 X 10^7 psi

Poisson's Ratio = 0.499

The stress distributions for the elastic (t = 0) and the steady state are shown in Figures 1 through 3 as a function of radius. For comparison, the results obtained by Greenbaum are also plotted. The agreement is excellent. These results show that due to creep, the elements with higher stresses are relieved of load which in turn is passed on to the elements with low stresses. Figure 4 gives the inside and outside redial deformation as a function of time. The steady state is reached after two hours. After two hours, the curves become straight lines and can be extrapolated to get the deformations at a larger time.

This extrapolation is possible only in cases where the creep rate is independent of time.

Elastic-plastic thick walled cylinder

In this problem, a thick cylinder subjected to an internal pressure is analyzed. At t = 0, there is a plastic zone in which the stresses exceed the yield strength of the material. The material is assumed to be Von Mises and Poisson's Ratio is assumed to be 0.3. The creep strain is assumed to be

$$\epsilon_e^c = 6.4 \times 10^{-18}\ \sigma_e^{4.4}\ t \tag{25}$$

In Figures 5 to 8, a is the inside diameter and b the outside. The stress distribution for this problem at t = 0, 0.55, 2.05, 4.93, and 9.01 hours are shown as function of radius in Figures 5 through 7. The distribution of these stresses at t = 0 matches with those calculated by Chang and Nair (1972). The other results are being presented for the first time and are not available in literature for comparison. It is interesting to note how the elements with

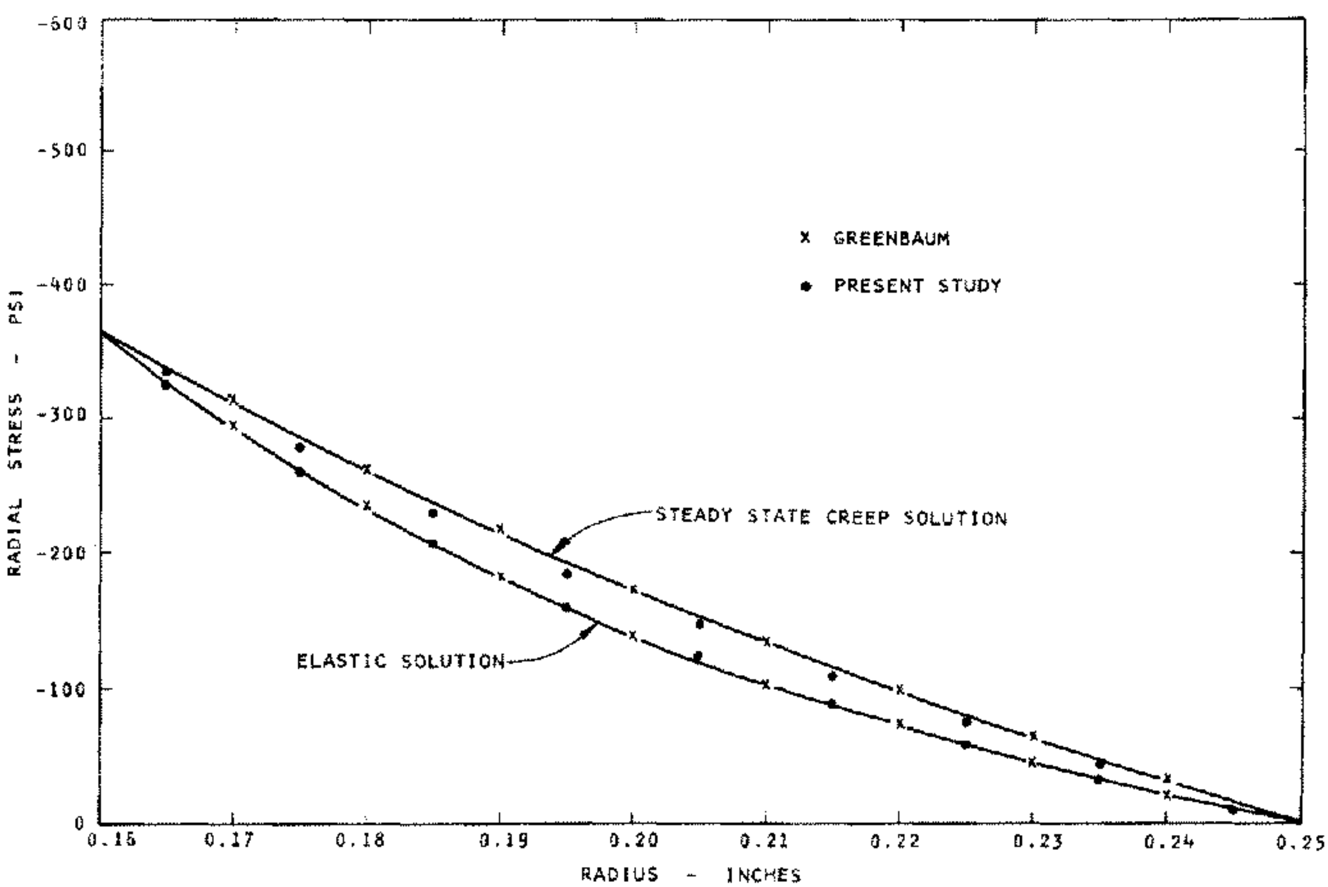

Figure 1. Radial Stress Distribution in a Thick Walled Cylinder.

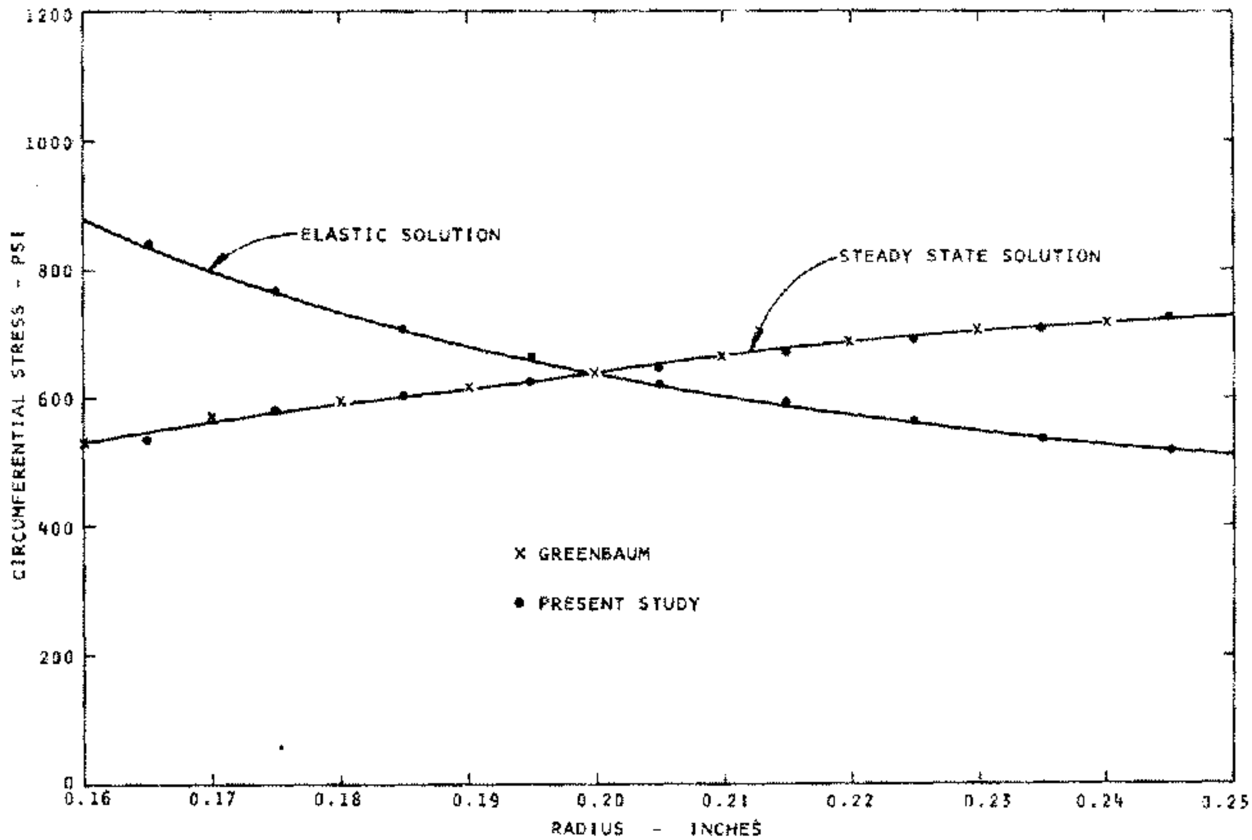

Figure 2. Distribution of Circumferential Stress in a Thick Walled Cylinder.

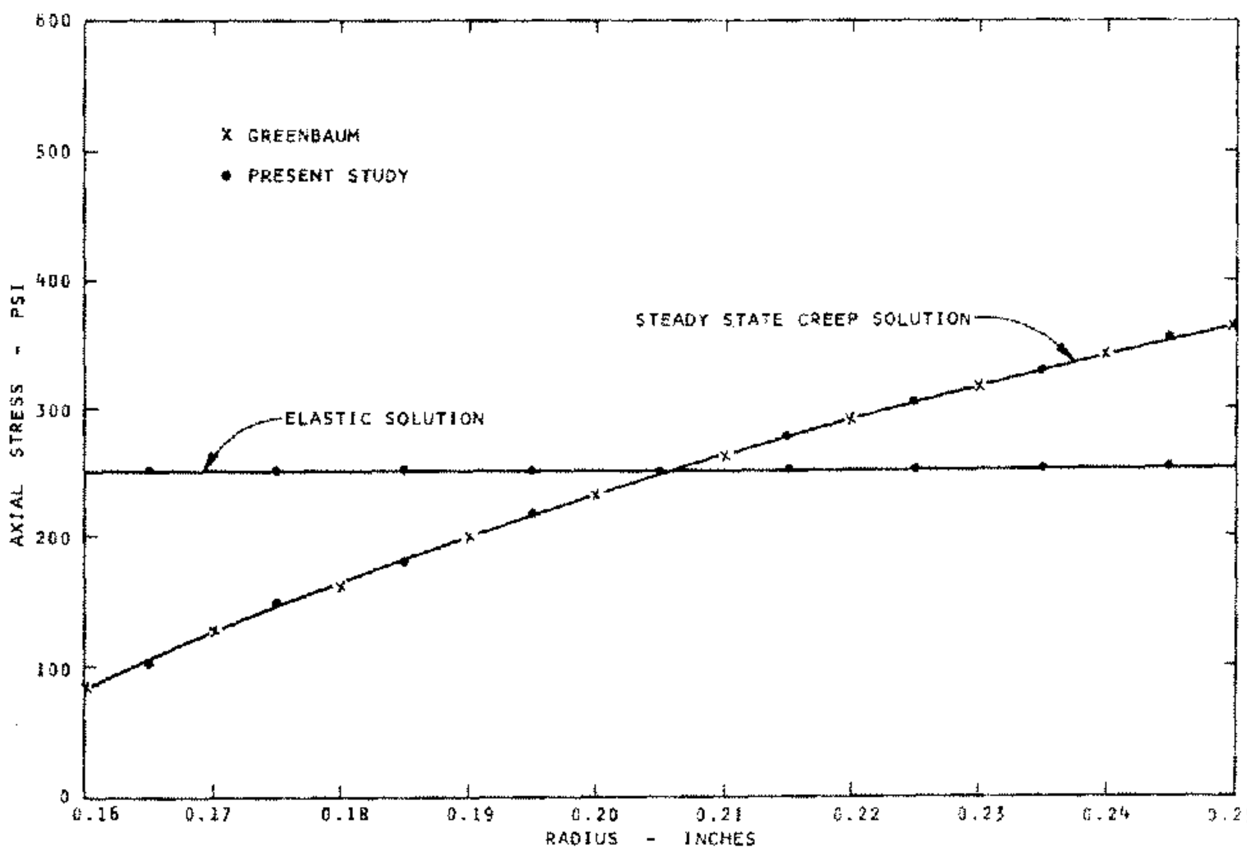

Figure 3. Axial Stress Distribution in a Thick Walled Cylinder.

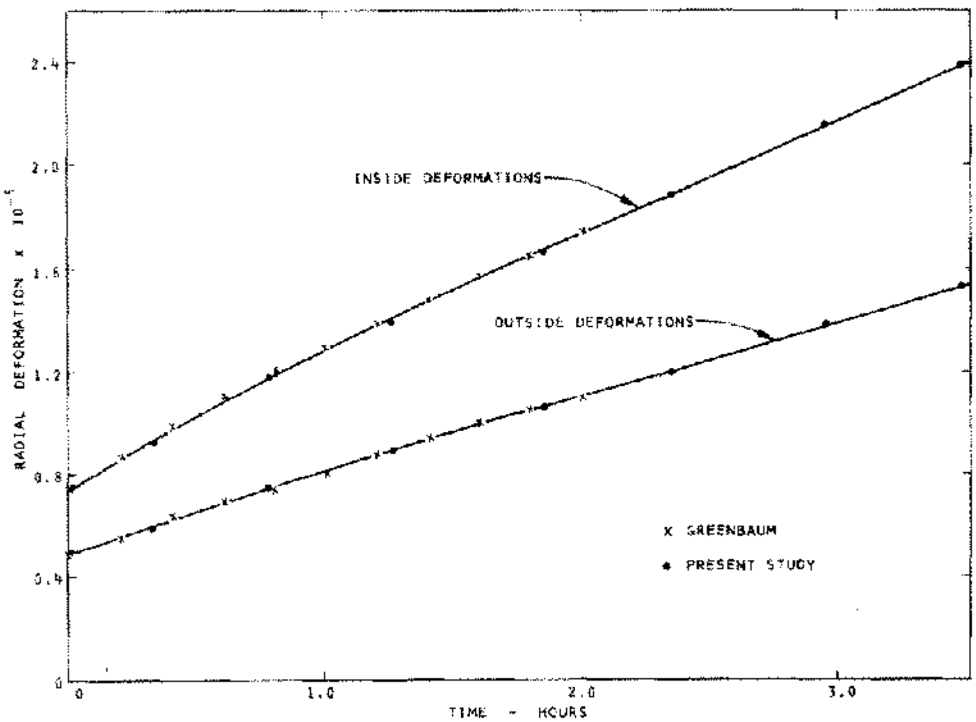

Figure 4. Inside and Outside Creep Deformation of a Thick Walled Cylinder.

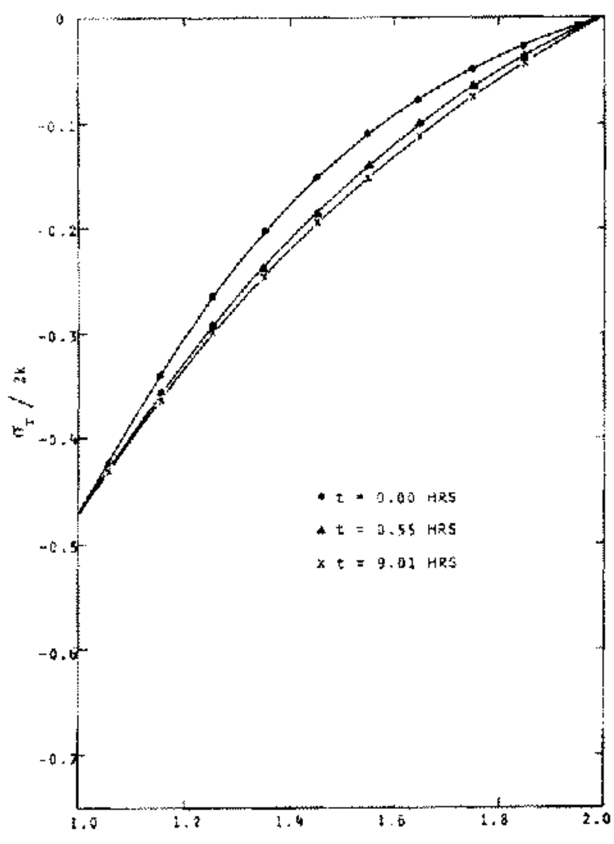

Figure 5. Distribution of Radial Stress with Time in a Thick Walled Pipe with Plastic Zone.

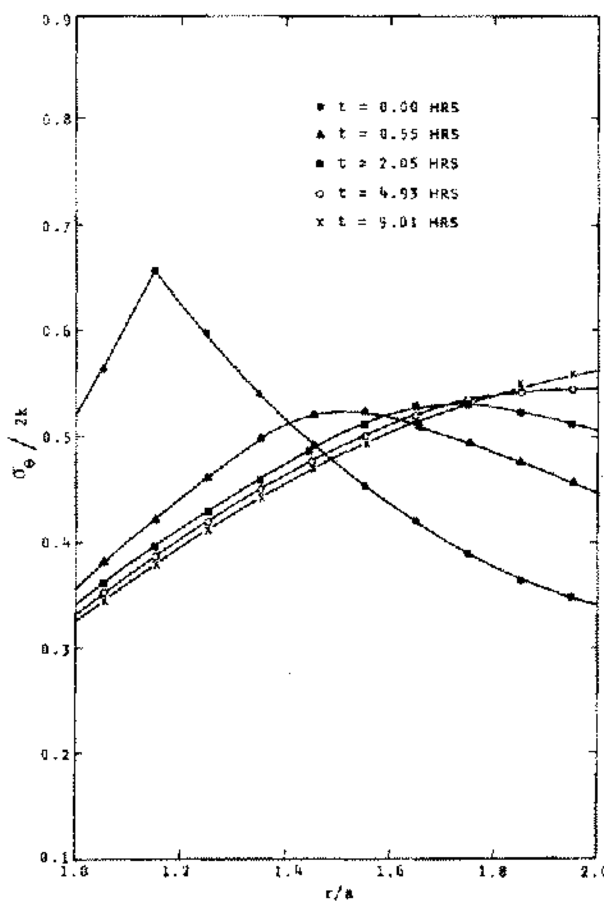

Figure 6. Distribution of Circumferential Stress with Time in a Thick Walled Pipe with Plastic Zone.

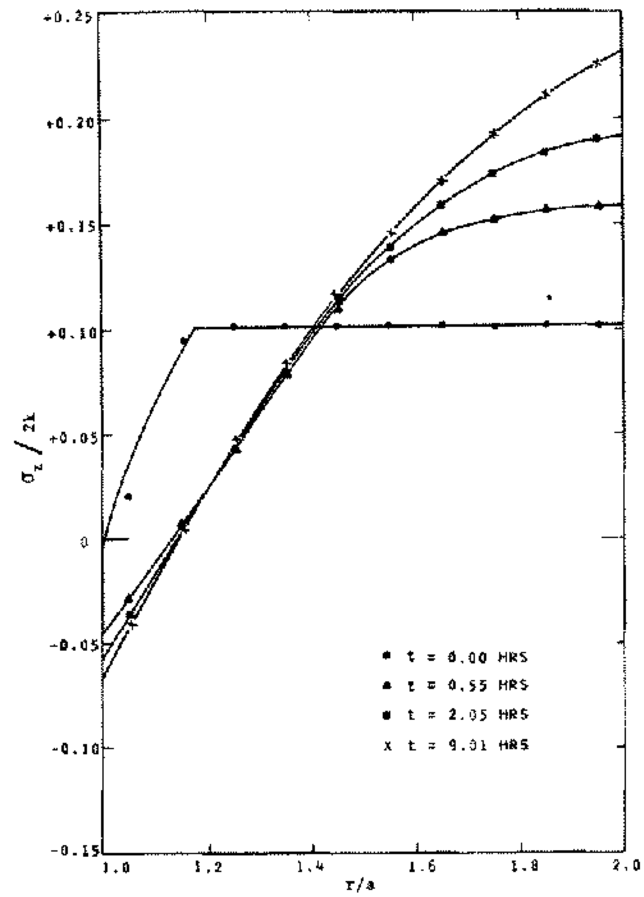

Figure 7. Distribution of Axial Stress with Time in a Thick Walled Pipe with Plastic Zone.

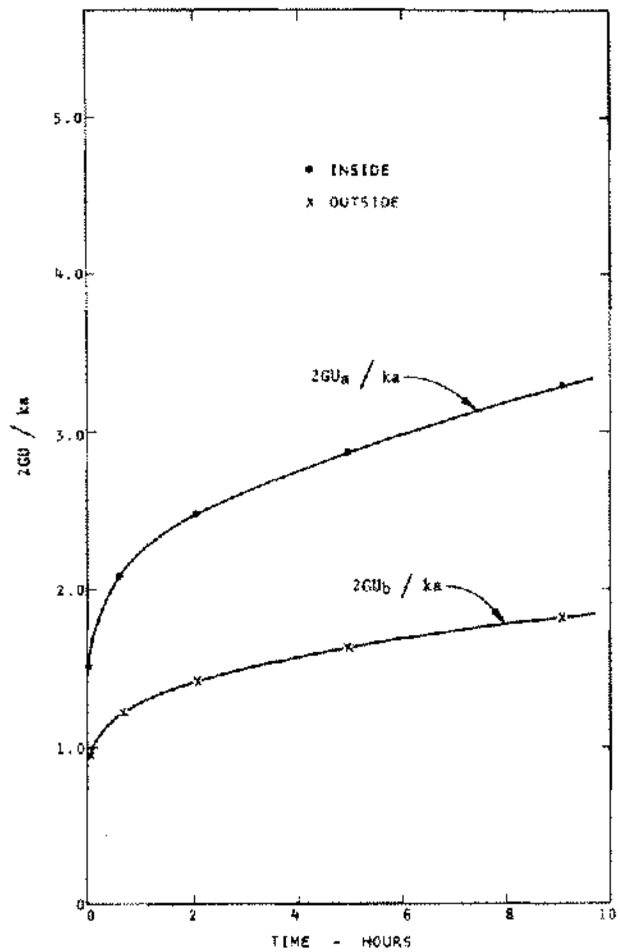

Figure 8. Inside and Outside Displacements with Time in a Thick Walled Pipe with Plastic Zone.

high circumferential stress relax and stress transferred to outside of the cylinder. Similar trend is shown for axial stress. However, as in previous examples, the radial stress increases in magnitude with creep.

In Figure 8, the inside and outside deformations have been plotted as a function of time. It is seen that these curves are very much like creep curves, which is to be expected. The steady state has been reached around six hours.

Spherical cavity deep in ground

To illustrate that the programs developed are very useful in the prediction of creep closure in case of cavities in rock salt, results of the analysis of a spherical cavity are presented in Figures 9 and 10. The spherical cavity of 95 feet in diameter was analyzed under a hydrostatic pressure of 3100 psi. The time hardening law was used. The material constants used are indicated on the figures.

The relaxation of the stresses and increase in displacement with time should be noted. It can be observed that the majority of stress relaxation occurs in the initial 24 hours and practically all of it is complete in one month. In evaluating failure through continuing deformation the possibility of a creep rupture should be investigated. The finite element method can be used to analyze cavities with liners or any other artificial support. Stresses can be calculated with time as load gets transferred to the supports with time. This gives us an important tool to design these supports rationally.

CONCLUSIONS AND SUMMARY

Finite element programs have been developed for the purpose of performing creep analysis for plane strain and axisymmetric boundary value problems. The materials can be no-tension and elastic-plastic materials. Joints and geological discontinuities may be present. No example has been presented having these discontinuities due to expensive nature of the program. The program can predict complete time history response to complicated structures. The empirical creep law used is capable of reproducing the highly non-linear response of materials like rock salt. The

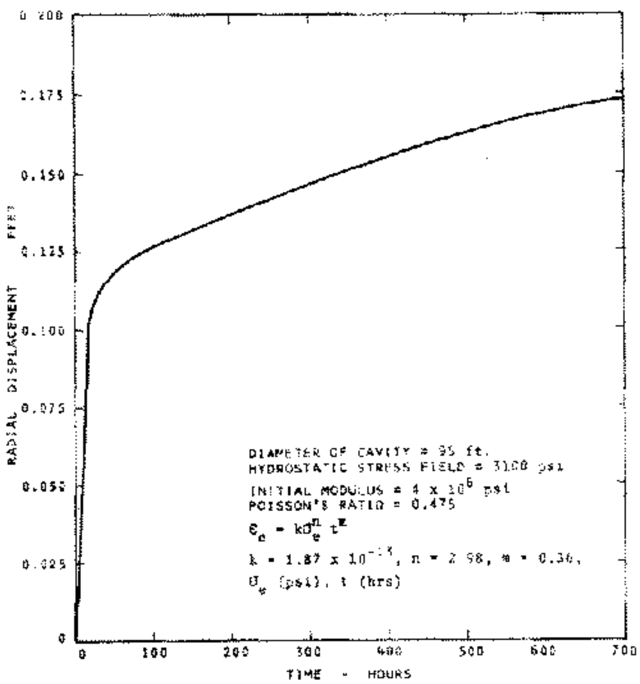

Figure 9. Radial Displacements at Face of Spherical Cavity.

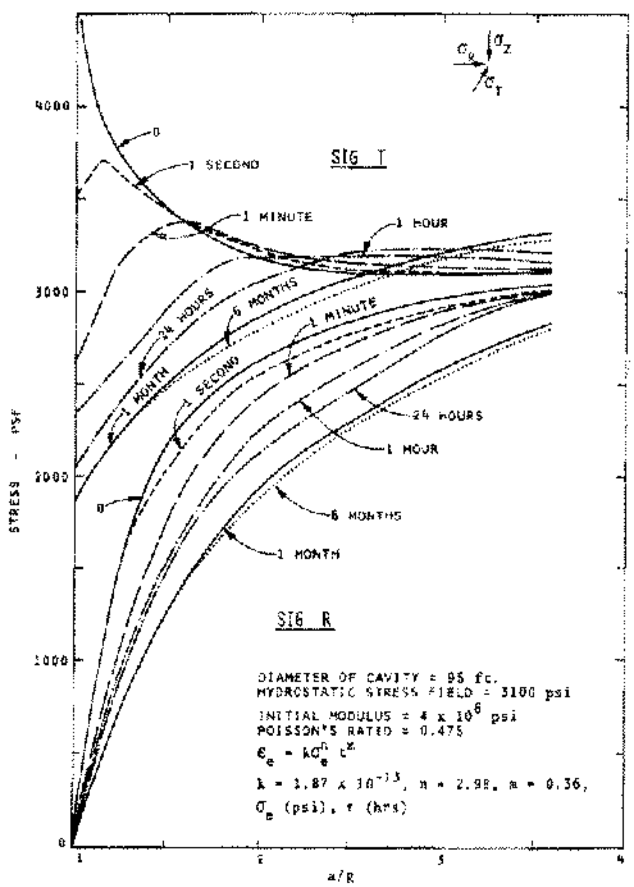

Figure 10. Stresses with Time for a Spherical Cavity.

creep parameters are easily evaluated from a simple compression creep test. The law is very general and permits consideration of strain hardening or age hardening laws. Any other empirical law can be easily incorporated in the programs.

At the present time, even after a lot of experience with the programs, it is premature to indicate with any degree of confidence which criterion for automatic time generation scheme will ensure stability of solution under all loadings and boundary value problems. A certain amount of experience and trial and error is necessary in solving practical problems. The examples demonstrate that the program is operational and can be used to solve boundary value problems with complex geometry and loadings.

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